Name____

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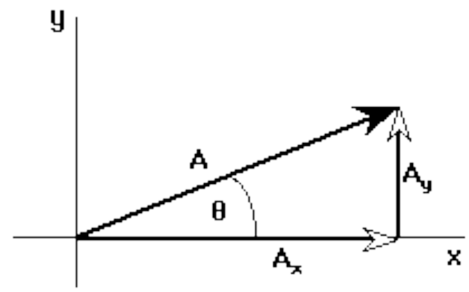
Free Particle Model Reading 2:
Vectors, Scalars, and Trigonometry

Some physical quantities like time, mass, and speed are characterized by having only a size, also referred to as magnitude. Such quantities are called **scalars**. Other physical quantities like displacement, velocity, acceleration, and force have both magnitude and direction. Such quantities are called **vectors**.

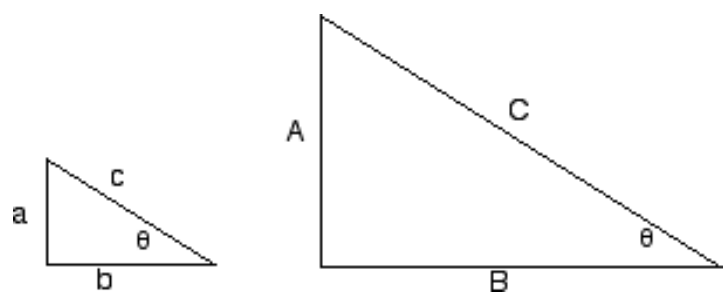
In the previous units, we have been able to indicate the direction of the velocity or acceleration by using positive and negative signs because we only considered motion along a line. Much of our study of forces will be two-dimensional, requiring us to be more specific when indicating direction. We must also be more careful when adding vectors in two dimensions. To perform a vector sum, we need to be able to take the vectors that aren't aligned with a coordinate axis and align them with the coordinate axes. Then, vectors along each axis can be added.



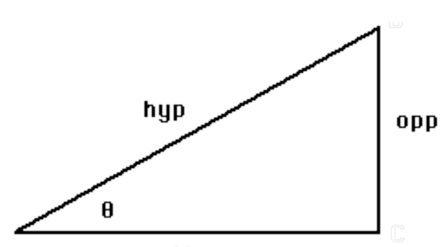
To align a vector with a coordinate axis, we represent an unaligned vector, such as **A** on the right, with two **component vectors**, such as Ax and Ay, whose sum is the same as the original vector. Each component vector is aligned along one of the coordinate axes and they are perpendicular to one another.

Again, since Ax and Ay are not in the same direction, adding their magnitudes will not give you the value of the magnitude of A. However, this is a right triangle, so you could use the Pythagorean theorem to relate the magnitudes of the sides.

Everything you need to know about trigonometry for this course:



Once upon a time, someone noted a very interesting property of similar right triangles. If you take the ratio of any two sides on the little triangle, such as a/b, the ratio is exactly the same as A/B for the big triangle. The value of the ratio corresponds to a specific angle, θ (theta). If A/B = 1, then θ = 45 degrees no matter how big or small the triangle is. If a/c = 1/2, then θ = 30 degrees no matter how big or small the triangle is. If someone would sit down and work out every pair of ratios for every shape of right triangle (which has been done), then when we knew the angle, we could use the value of the ratio between two sides to find the length of an unknown side. For example, suppose we know θ is 30 degrees and A is 10 meters long. Since the ratio of A/C is 1/2 for a 30 degree right triangle, we know that C must be 20 meters long.



To formalize what has just been said, let us first name each of the sides relative to angle θ. The side next to the angle is called the **adjacent** side, the one across the triangle from the side is called the **opposite** side, and the longest side of a right triangle is called the **hypotenuse**.

Our next bit of formalizing involves naming the ratios of the sides. Although we could name six, we will just use three, since the other three are reciprocals of the first three.
Sine of angle θ is the ratio of the opposite side divided by the hypotenuse.
Cosine of angle θ is the ratio of the adjacent side divided by the hypotenuse.
Tangent of angle θ is the ratio of the opposite side divided by the adjacent side.

Shorthand notation: $\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$

So whenever you see sin(45) or tan(21) just remember that these are just numbers; sin(45) is a decimal number that results from diving the opposite side by the hypotenuse in a 45-45-90 triangle. You can get the value of the ratio from your calculator or from a trigonometric table. If you use your calculator, be sure that it is in degree mode.

To find the angle when the sides of a triangle are known, the inverse trigonometric relations are used. In other words, given a ratio, what is the corresponding angle? On a trigonometric table this is easy to see. If the tangent ratio for a triangle is 0.8, then we can look in the tangent column until we find a value of 0.8 and then look over to see that the corresponding angle is 39 degrees. On your calculator, you would hit "inverse tangent" which is "2nd" "tan" on most calculators and then enter the ratio. The calculator will then display the angle.

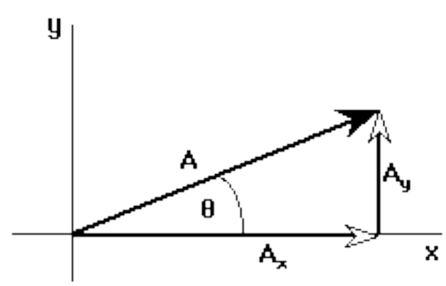
Shorthand notation: $\sin^{-1}\left(\frac{opp}{hyp}\right) = \theta \quad \cos^{-1}\left(\frac{adj}{hyp}\right) = \theta \quad \tan^{-1}\left(\frac{opp}{adj}\right) = \theta$

(Note that the -1 is not an exponent meaning "take the reciprocal" in this case. Instead it means, "find the angle that corresponds to this ratio of triangle sides.")

Example:
In order to find the component vectors, A_x and A_y, we can use the trigonometric relations we have just identified.

$$\cos\theta = \frac{A_x}{A} \qquad \sin\theta = \frac{A_y}{A}$$

$$A_x = A\cos\theta \qquad A_y = A\sin\theta$$



Now we could plug in the magnitude for vector **A** and the angle for θ. Taking the sine and cosine of θ will give us a decimal in each case that we can multiply with the magnitude of vector **A**.

If we already knew the values of A_x and A_y, we could use inverse tangent to find the angle.

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

More examples can be found in the practice problems portion of the class website.